Effects of a pulsed electromagnetic field on the impact response of electrically conductive composite plates

1. Introduction

Electromagnetic solids encompass a broad class of materials in which the interaction between mechanical and electromagnetic loads has a pronounced effect on the deformed state. Recently, such multifunctional materials have been employed in a wide range of applications due to their unique properties. The main focus of this work is on the effects of an electromagnetic field on the dynamic mechanical response of electrically conductive anisotropic composites. In particular, we are interested in carbon fiber polymer matrix composites that play a pivotal role in a number of industries, including aerospace and automotive. In the context of impact, the primary reason that made electrically conductive composites attractive for researchers is the possibilities for improvement of impact resistance of structures by applying electromagnetic loads.

In the most general case, behavior of electromagnetic materials is described by a nonlinear system of equations of motion and Maxwell’s equations. Due to the complexity of the physical phenomenon as well as insuperable mathematical difficulties associated with simultaneous solving of generally nonlinear system of equations of motion and Maxwell’s equations, there are only a few accomplished solutions, even to the problems of three-dimensional linear isotropic elasticity with electromagnetic effects. The situation is better for isotropic plates and shells where a number of solutions for coupled problems exist [1-6]. In recent work [7,8], the effects of steady and slowly-varying electromagnetic fields on the mechanical response of composite plates were studied. It has been shown that an electromagnetic field, depending on the direction of its application and intensity, may significantly enhance or reduce the stress state of the mechanically loaded composite plate.
On the other hand, an electromagnetic field in electrical conductors is also manifested by Joule heating which may have an adverse effect on the structure. For instance, it has been shown in the literature [9] that application of even moderate DC currents to the composites results in a considerable rise of the thermal stresses. These stresses seem to be avoidable if electromagnetic loads of high intensity but short duration (i.e. pulsed electromagnetic loads) are applied.

The present work aims to study the effects of pulsed electromagnetic fields on the impact response of carbon fiber polymer matrix composites and investigate possibilities for stress mitigation in a mechanically loaded composite by concurrent application of an electromagnetic field.

2. Governing Equations and Solution Procedure

In solids with electromagnetic effects, the interaction between mechanical and electromagnetic fields is due to the Lorentz ponderomotive force that is exerted by the electromagnetic field. Analysis of this interaction of fields requires simultaneous solving of Maxwell’s equations for electromagnetic field [7,8,10-12],

\[
\begin{align*}
\text{div} \mathbf{D} &= \rho_e, \quad \text{curl} \mathbf{E} = -\partial \mathbf{B} / \partial t, \\
\text{div} \mathbf{B} &= 0, \quad \text{curl} \mathbf{H} = \mathbf{j} + \partial \mathbf{D} / \partial t,
\end{align*}
\] (1)

and equations of motion of continuous media,

\[
\frac{\partial \tau_{ij}}{\partial x_j} + \rho \left( F_i + F_i^L \right) = \rho \frac{\partial^2 u_i}{\partial t^2}.
\] (2)

Here \( \mathbf{E} \) and \( \mathbf{D} \) are the electric field and electric displacement vectors, \( \mathbf{B} \) and \( \mathbf{H} \) are the magnetic induction and magnetic field vectors, \( \rho_e \) is the charge density, \( \mathbf{j} \) is the electric current
density vector, $\tau_{ij}$ are the components of the mechanical stress tenor, $F_i$ are the components of the body force per unit mass, $F_i^L$ are the components of the Lorentz force per unit mass, $u_i$ are the displacement components, $\rho$ is the material density of solid body, $x_j$ are the coordinates, and $t$ is time. It has been shown in the literature [13] that in the case of an electrically anisotropic and linear but magnetically isotropic solid body (which is the case for carbon fiber reinforced polymer matrix composites), the Lorentz ponderomotive force can be written in the form

$$F^L = \rho \left( E + \frac{\partial u}{\partial t} \times B \right) + \left( \sigma \cdot \left( E + \frac{\partial u}{\partial t} \times B \right) \right) \times B + \left( \left( \varepsilon - \varepsilon_0 \cdot I \right) \cdot E \right) \times B \nabla \left( \frac{\partial u}{\partial t} \right),$$

where $\sigma$ is the electrical conductivity tensor, $\varepsilon$ is the electrical permittivity tensor, $\varepsilon_0$ is the vacuum permittivity, $J$ is the density of the external electric field, $I$ is the unit tensor of second order, $\nabla$ is the gradient operator, and Einstein’s summation convention is adopted with respect to the index $\alpha$. Therefore, the Lorentz force, $F^L$, which enters the equations of motion as a body force, makes the system of governing equations (1) and (2) coupled and nonlinear.

In the case of thin plates, three-dimensional (3D) Maxwell’s equations (1) and equations of motion (2) may be reduced to 2D equations by means of the classic Kirchhoff hypothesis of nondeformable normals and the corresponding electromagnetic hypotheses introduced by Ambartsumyan et al. [1]. In the hypothesis proposed by Ambartsumyan et al., it is assumed that the tangential components of the electric field vector and the normal component of the magnetic field vector do not change across the thickness of the plate. Moreover, a 2D (quasi-static) approximation to Maxwell’s equations is derived by representing electromagnetic field functions via series expansions with respect to the thickness coordinate and integrating 3D Maxwell’s
equations across the thickness of the plate. The resulting 2D system of equations of motion and Maxwell’s equations is a nonlinear mixed system of parabolic and hyperbolic partial differential equations in the form

\[
\frac{\partial \mathbf{g}}{\partial y} = \Phi \left( y, t, \mathbf{g}, \frac{\partial \mathbf{g}}{\partial t}, \frac{\partial^2 \mathbf{g}}{\partial t^2} \right),
\]

where the unknown \( N \)-dimensional vector \( \mathbf{g}(y,t) \) comprises the unknown stress and moment resultants and the electric field and magnetic induction components, as well as the middle plane displacements and their first derivatives.

The numerical solution procedure for the system of governing equations (4) used in this work consists in the sequential application of finite difference time and spatial integration schemes, quasi-linearization, and a finite difference spatial integration of the obtained two-point boundary-value problem. In this study, the Newmark finite difference time integration scheme [14] is used, followed by the finite difference space integration with respect to one of the spatial coordinates (e.g., the \( x \)-coordinate). This yields a system of nonlinear ODEs that together with boundary conditions at the edges of the plate forms a nonlinear two-point boundary-value problem. To solve this system, a quasilinearization method of Bellman and Kalaba [15] is applied to the system (4) and accompanying boundary conditions at the moment of time \( t + \Delta t \). A sequence vector \( \{ \mathbf{g}^{k+1} \} \) is generated by the linear equations

\[
\frac{d\mathbf{g}^{k+1}}{dy} = \Phi(\mathbf{g}^{k}) + J(\mathbf{g}^{k})(\mathbf{g}^{k+1} - \mathbf{g}^{k}),
\]

and the linearized boundary conditions

\[
\frac{\partial \mathbf{g}^{k+1}}{\partial y} = \Phi(\mathbf{g}^{k}) + J(\mathbf{g}^{k})(\mathbf{g}^{k+1} - \mathbf{g}^{k}),
\]
with \( g^0 \) being an initial guess. Here \( g^k \) and \( g^{k+1} \) are the solutions at the \( k \)-th and \( (k+1) \)-th iterations, and \( J(g^k) \) is the Jacobian matrix defined as

\[
J(g^k) = \begin{bmatrix}
\frac{\partial \Phi}{\partial g_1}(g^k) \\
\vdots \\
\frac{\partial \Phi}{\partial g_J}(g^k)
\end{bmatrix}.
\]

The Jacobian matrix needs to be calculated analytically. A sequence of solutions \( \{g^{k+1}\} \) of the linear systems (5) ultimately converges to the solution of the nonlinear system (4). The iterative process is terminated when the desired accuracy of the solution is achieved.

In this study, we use the superposition method together with the stable discrete orthonormalization technique to solve the linear two-point boundary-value problem (5)-(6) \cite{7,8,16,17}. We seek the solution to the boundary-value problem (5)-(6) at the \( (k+1) \)-th iteration in the form of a linear combination of \( J \) linearly independent general solutions and one particular solution

\[
g^{k+1}(y,t + \Delta t) = \sum_{j=1}^{J} c_j G^j(y,t + \Delta t) + G^{J+1}(y,t + \Delta t),
\]

where \( G^j, j = 1,2,3,...,J \) are solutions of the Cauchy problem for the homogeneous system (5)-(6) and \( G^{J+1} \) is the solution of the Cauchy problem for the inhomogeneous system. Construction of the solution in the form (8) via a straightforward integration will not yield satisfactory results since the matrix of the system (5) in the problems with mechanical and electromagnetic effects is
usually “ill-conditioned,” which leads to the loss of linear independency in the solution vectors \( G^j, j = 1, 2, 3, ..., J + 1 \). One of the suggested methods to circumvent the loss of linear independency in the solution vectors is to apply an orthonormalization procedure [16-18]. In this study, a modified Gram-Schmidt orthonormalization procedure is adopted due to its numerical stability, relative simplicity, and modest computational requirements.

3. Dynamic Response of a Composite Plate Subjected to Pulsed and Impact Loads

Consider a thin unidirectional fiber-reinforced electrically conductive composite plate of width \( a \) and thickness \( h \) subjected to the pulsed electric current of density \( J^* \) and the transverse short duration load \( P \) and immersed in the magnetic field with the induction \( B^* \). The plate is transversely isotropic, where \( y-z \) is the plane of isotropy and \( x-y \) coincides with the middle plane of the plate. The plate is assumed to be long in the direction of the applied current (\( x \)-direction), simply supported along the long sides, and arbitrarily supported along the short sides (Fig. 1). The plate is subjected to a short duration line transverse load, which results in the time-varying compressive pressure distribution, \( P(y,t) \), given by:

\[
P(y,t) = \begin{cases} 
\frac{B_{zz} b}{R} \sqrt{1 - \left(\frac{y}{b}\right)^2} \sin \frac{\pi t}{\tau_p}, & -b \leq y \leq b, \quad 0 < t \leq \tau_p, \\
0, & \frac{a}{2} \leq y < -b, \quad b < y \leq \frac{a}{2}, \quad t > \tau_p.
\end{cases}
\]

(9)

This line pressure mimics the Hertz contact pressure distribution in the quasi-static problem of the elastic impact. Here \( b \) is the half-size of the contact zone; \( R \) is the radius of the impactor; \( \tau_p \) is the impact characteristic time parameter, which determines duration of the applied
pressure; and $B_{22}$ is defined as $B_{22} = E_y \left(1 - \nu_{xy} \nu_{yx} \right)$, where $E_y$ is Young’s modulus for the isotropy plane, $\nu_{xy}$ is Poisson’s ratio characterizing contraction in the plane of isotropy due to forces in the direction perpendicular to it, and $\nu_{yx}$ is Poisson’s ratio characterizing contraction in the direction perpendicular to the plane of isotropy due to forces within the plane of isotropy. The load is assumed to result only in elastic deformation, and the plate is assumed to be initially at rest. In addition, the plate is carrying the pulsed current in the fiber direction and is immersed in the constant in-plane magnetic field as shown in Fig. 1. The density of the applied electric current and the magnitude of the constant magnetic induction are:

$$
J^* = \left( J^*_x, 0, 0 \right), \quad J^*_x(t) = J_0 e^{-t/\tau_c} \sin(\pi t / \tau_c), \quad t \geq 0,
$$

$$
B^* = \left( 0, B^*_y, 0 \right), \quad B^*_y = \text{const}.
$$

where $J_0$ is constant and $\tau_c$ is the characteristic time of the pulse current. As for the boundary conditions, the plate is simply supported:

$$
\tau_{zz}\bigg|_{y = \frac{h}{2}} = -P(y, t), \quad u_y\bigg|_{y = \pm \frac{a}{2}} = 0, \quad M_y\bigg|_{y = \pm \frac{a}{2}} = 0.
$$

and the boundary conditions for the electromagnetic field are taken as

$$
\left( E_x - \frac{\partial w}{\partial t} B^*_y + \frac{\partial v}{\partial t} B^*_z \right)\bigg|_{y = \pm \frac{a}{2}} = 0, \quad E_x\bigg|_{y = \frac{a}{2}} = 0.
$$

The first electromagnetic boundary condition ensures that electric current does not pass through the boundary $y = -a/2$. The formulation and numerical procedure developed in the previous section is employed here to investigate the dynamic response of the plate under electromagnetic
pulsed and impact loading. In this case, the vector of unknowns for the boundary-value problem (5)-(6) is

\[ \mathbf{g} = \left[ v, w, W, N_{yy}, N_{yz}, M_y, E_x, B_z \right]^T. \]  

(13)

The following section provides results and discussion of the numerical study. In particular, we are interested in how the stress state in an impacted composite can be reduced by concurrent application of a pulsed electromagnetic field.

4. Results and Discussion

A code in FORTRAN was developed for the numerical analysis of the boundary-value problem (5)-(6). The accuracy and efficiency of the developed code was verified using analytical solution [19] for the vibration problem under a uniform constant mechanical load.

Computations have been performed for a long thin rectangular transversely isotropic plate with \( a = 0.1524 \) m and \( h = 0.0021 \) m subjected to impact (9) and electromagnetic loading (10). The plate is made of the AS4/3501-6 unidirectional carbon fiber reinforced polymer matrix composites with 60% fiber volume fraction. The material properties are: density \( \rho = 1594 \) Kg/m\(^3\), elastic modulus in fiber direction \( E_x = 102.97 \) GPa, elastic modulus in transverse direction \( E_y = 7.55 \) GPa, Poisson’s ratios \( \nu_{xx} = \nu_{yz} = 0.3 \), electric conductivity in fiber direction \( \sigma_x = 39000 \) 1/Ω\(^{-1}\)m, and electrical permittivity in fiber direction \( \varepsilon_x = 2.501502912 \times 10^{-10} \) F/m.
Fig. 2 shows the profiles of the impact load, Eq. (9) with $\tau_\rho = 10\,\text{ms}$, $b = h/10000$ and $R = 6h$, and the pulsed current, Eq. (10) with $\tau_c = 10\,\text{ms}$ and $J_0 = 10^6\,\text{A/m}^2$, as functions of time. Note that peaks in electric pulse and impact load occur at the same time for both current pulse and impact load. We investigate the deformation of the plate for different values and directions of the current density, magnetic induction, and pulse duration. Fig. 3 shows the transverse deflection of the plate in the middle point $y = 0$ as a function of time for three different loadings. As one can see, when the electromagnetic load is applied, the reduction in the amplitude of the vibrations is noticeable. This fact is very desirable in engineering applications since the impact damage can be decreased or even eliminated by applying proper electromagnetic load to the solid body.

Fig. 4 and Fig. 5 show the effects of the magnitude of the pulsed current and the external magnetic induction on the impact response of the plate. As shown in Fig. 4, the magnitude and direction of the electric current and magnetic induction have a significant effect on the response of the plate. For instance, an increase in the current density may significantly reduce deflection of the plate and have a “stiffening effect” on the plate response to impact load. However, if the magnitude of the current is too large, it will become the dominant load and after repressing the effect of the impact, it will itself produce large deflections that may be more harmful than the impact. Similar results are shown in Fig. 5 for the effect of the external magnetic field on the impact behavior of the plate. These results show that there is an optimum for the electromagnetic load in improvement of the impact response. Increasing pulse or external induction magnitudes are not always effective.

The effect of the characteristic time of the pulsed current on the impact response is presented in Fig. 6. It can be seen that when the characteristic time of the current is large, e.g., $\tau_c = 100\,\text{ms}$,
the dynamic response of the plate is almost the same as the one with pure impact loading and the electromagnetic load has a little effect. For a small characteristic time, say $\tau_c = 5\text{ ms}$, little change in frequency is observed for the selected values of current and magnetic induction. When the characteristic time of the applied electric pulse is $\tau_c = 10\text{ ms}$, which is the same as time of the application of the impact load, reduction in the transverse deflection is observed.

5. Conclusions

The impact response of a long transversely isotropic plate in the presence of an electromagnetic field was studied. Governing equations describing electromagnetic and mechanical field interactions in anisotropic materials and a corresponding two-dimensional approximation for transversely isotropic plates are discussed. A numerical solution procedure for solving a nonlinear mixed system of parabolic and hyperbolic PDEs for the plates with electromagnetic effects is developed and applied to the problem of the mechanical response of a long transversely isotropic current-carrying plate subjected to impact and pulsed electromagnetic loads.

The numerical results show that electromagnetic loading changes the pattern and amplitude of the transverse vibrations of the impacted plate. The plate deformation depends on both magnitude and direction of the electric current and magnetic induction. Furthermore, application of an appropriate combination of the pulsed current and magnetic field can significantly reduce the vibration of the plate due to impact. The results show that there is an optimum combination of the pulsed electric current and magnetic field, which leads to considerable suppression of the plate vibrations.
Figures

**Figure 1.** Unidirectional composite plate subjected to pulsed electromagnetic current and transverse impact load

**Figure 2.** Profiles of the impact load with $b = h/10000$ and $R = 6h$, and the pulsed current with $J_0 = 10^6 \text{ A/m}^2$ ($\tau_p = \tau_c = 10 \text{ ms}$)

**Figure 3.** Transverse deflection of the center of the plate as a function of time for different loadings
Figure 4. Effect of the magnitude of the pulsed current on the impact response of the center of the plate

Figure 5. Effect of the magnitude of the magnetic field on the impact response of the center of the plate

Figure 6. Effect of the characteristic time of the pulsed current on the impact response of the plate
References


