Introduction

In recent years, functionally graded materials (FGMs) have attracted much interest in a wide range of engineering applications, such as aerospace, automotive, manufacturing and medical, to name a few. The name “FGMs”, coined by a group of Japanese material scientists in the early 1980s [1], refers to nonhomogeneous composite materials composed of two or more phases. Unlike other composites such as laminated plates, properties are continuously variable through the thickness and/or in the in-plane directions in FGMs. The continuous varying of material properties allows for enhancement of properties such as, but not limited to, mechanical, thermal, electrical, or combinations of the three. Additionally, because properties are continuous, abrupt transitions in properties are not present. This smooth transition of properties can reduce thermal and residual stresses, as well as stress concentrations found in laminated or bonded types of composites.

The most appealing aspect of FGMs is the ability to tailor material properties to specific demands. Recently, interest and research have been focused on FGMs as replacements for traditional thermal barriers in aerospace applications. Thermal barriers serve to protect the underlying material from effects of elevated temperatures. Traditional thermal barriers rely on bonded, coated, or laminated protection layers over exterior panels. Popular materials for the outer layer are typically ceramic-based due to ceramic’s low thermal conductivity. Take for example the NASA space shuttle; ceramic tiles are bonded to the vehicle’s outer skin to protect it from the extreme temperatures of re-entry into the Earth’s atmosphere. However, an abrupt
change in thermal expansion and stiffness make these tiles susceptible to cracking and debonding. Additionally, the high density of the ceramic tiles adds significant weight to the vehicle. Here-in lies an essential advantage of an FGM: the continuous variation in material properties can alleviate thermal stress and stress concentrations found in a traditional thermal barrier coating (TBC) as well as reduce the overall weight. Furthermore, it can be seen that FGMs can serve as dual purpose roles as a structural member as well as thermal barrier, thus eliminating the need of a dedicated thermal barrier.

Past studies have investigated multifunctional FGMs as thermal barrier in aerospace applications by varying the through-thickness and/or in-plane directions of material properties [3, 4]. Here the structural components of the airframe perform an additional role as a thermal barrier in the presence of elevated temperatures. Some studies [4-6] consider composite ceramics and a metal matrix approach to dealing with the thermal load. The advantage of these types of FGMs is the considerably low thermal conductivity of ceramics, but it comes at the expense of high density. Additionally, these ceramics have low fracture strength, making them susceptible to cracking and debonding.

Other approaches make use of a Titanium Boride/Titanium FGM to tailor the thermo-mechanical response of the structure [3]. In this work, the high stiffness-to-weight ratio of Titanium boride along with the lower thermal conductivity of the Titanium is used to stiffen the structure and provide an enhanced thermo-mechanical response.

The task of this project is to effectively model the mechanical and thermal properties of FGMs for use in thermal barriers and to select materials well suited for such applications. Items to consider in the selection and analysis are maximizing service temperature, chemical stability,
structural integrity, and maximizing strength-to-weight ratio. In addition, modeling of the thermo-mechanical response is investigated by use of finite element methods. These methods are used to clearly show the advantage of using a FGM over a traditional thermal barrier.

**Problem Formulation**

Because of the direct correlation between air speed and temperature and an increasing desire for air travel into the high supersonic and hypersonic regimes, the effects of temperature, pressure, acoustic shock, and chemically reacting airflows play a predominant important role in the design of such in-flight structures [2]. It is well known that as air speed increases (Mach number), temperatures increase as a result of friction between the air flow and the vehicle [2, 7]. As a result, a thermal barrier is needed to protect the underlying structure from the extreme temperatures. The correlation of Mach number and temperature on different airframe components can be seen in Figure 1. In this work, slightly curved thin rectangular plates are subjected to an elevated temperature on the outer skin and a lower internal temperature. As a result of the large temperature difference through a thin cross section, a steep temperature gradient is introduced, creating non-uniform thermal stress due to mechanical constraints.

**Solution Procedure**

Because material properties of an FGM vary continuously, the equivalent material properties need to be defined. It is assumed that the ceramic volume fraction can be represented as an exponential function of the thickness coordinate, \( z \), [5] by
\[ v_c(z) = \left( \frac{z}{h} + \frac{1}{2} \right)^n, \quad -\frac{h}{2} \leq z \leq \frac{h}{2} \]  

(1)

here \( h \) is the thickness of the structure, and \( n \) is a volume fraction exponent. It can be seen that large values on \( n \) create a metal-dominant FGM, while small values create a ceramic-dominant FGM. A visual representation of the material grading is shown in Figure 2. The material properties of the FGM are estimated using Voigt’s upper and Reuss’s lower bounds [8-9] given by

\[ P(z)_{\text{Voight}} = P_1 v_1 + P_2 v_2 \]  

(2)

\[ P(z)_{\text{Reuss}} = \frac{P_1 P_2}{P_1 v_2 + P_2 v_1} \]  

(3)

where \( P_1 \) and \( P_2 \) are the material properties of the two constituent materials, and \( v_1 \) and \( v_2 \) are the volume fractions of the constituents. A conservative approach is used in determining the equivalent material properties by using the Voigt and Reuss bounds. For example, the lower bound is used when estimating the stiffness (Young’s modulus) of the FGM. This is in contrast to the upper bound for the thermal expansion, thermal conductivity, and Poisson’s ratio. The thermo-mechanical response of the behavior is performed sequentially. First the temperature of layer is determined using

\[ \nabla(k \nabla T) + Q = c_p \rho \frac{\partial T}{\partial t} + h(T - T_\infty) + \varepsilon \sigma (T^4 + T^4_\infty) \]  

(4)

here \( k \) is the thermal conductivity, \( Q \) is the heat load, \( T \) is the temperature, \( T_\infty \) is the ambient temperature, \( c_p \) is the specific heat capacity, \( \rho \) is the density, \( h \) is the heat convection coefficient, \( \varepsilon \) is the emissivity, and \( \sigma \) is the Stefan-Boltzman constant. All of the aforementioned material properties can be functions of the spatial coordinates and
temperature resulting in a nonlinear heat transfer. Second the mechanical response is
determined by the equation of motion

$$\frac{\partial \tau_{ij}}{\partial x_j} + \rho F_i = \rho \frac{\partial^2 u_i}{\partial t^2}$$  \hspace{1cm} (5)

here $\tau_{ij}$ is the stress tensor, $F_i$ is the body force per unit volume, $\rho$ is the density, and $u_i$ is the
displacement. The stress tensor can be further expanded as shown

$$\tau_{ij} = C(\varepsilon_{ij} + \alpha \Delta T)$$  \hspace{1cm} (6)

here $C$ is the stiffness tensor, $\varepsilon_{ij}$ is the mechanical strain, $\alpha$ is the thermal expansion coefficient,
and $\Delta T$ is the change in temperature. Similarly, the stiffness tensor and thermal expansion can
be functions of the spatial coordinates and temperature. Due to the dependence on
temperature for the mechanical properties, a coupled thermo-mechanical response is
produced. The problem is solved using commercially available finite element analysis code
(Abaqus). The following assumptions are made to simplify the problem for preliminary analysis:
1) thermal analysis is performed at steady state conditions, 2) Material properties are
temperature independent in the range of interest, 3) surface temperatures are uniform, 4) the
body is free of radiation, convection and conduction, 5) Mechanical analysis is performed at
static elastic condition with no initial strain.

Changes in the through-thickness material properties of the FGM panel are modeled by
discretizing the thickness into nine homogenous slices. Nine slices are used in this case because
this is the maximum allowable temperature degrees of freedom (19) available by the solver. To
solve the coupled field equations (2) and (3), Mindlin plate theory along with finite element
analysis is used.
Numerical Results

A computational study has been performed for the following FGM plate. The plate is 846x282 mm and 2.8 mm thick, with an aspect ratio (width/thickness) of 101 (thin plate). The outside surface is exposed to a uniform temperature of 500° C, while the inside is held at 20° C. The long edges are pinned along the top and bottom edges. The analysis is focused on the investigation of the effect of implementing FGMs compared to Aluminum with a traditional thermal barrier coating. Material properties for Aluminum and TBC (Zirconium) can be seen in Table 1.

The analysis has been completed for pure Aluminum, Zirconium, traditional TBC (1 mm thick), and FGM with n=0.2, n=2.0, and n=5.0. Here, Aluminum is used solely as a reference as alone the thermal load exceeds its recommended service temperature; the traditional TBC is used as the benchmark. Distribution of the ceramic volume fraction of the FGM through the thickness using (1) with varying values of $n$ can be seen in Figure 3. The weight of the panel for each case can be seen in Table 2. Using the traditional TBC as the benchmark, it is shown that for an FGM with $n=0.2$, a weight gain of 6.6% results, while a reduction of 19.8% and 29% results for FGMs with $n=2$ and 5, respectively.

The computed temperature profiles for each case are shown in Figure 4. As can be seen in Figure 3, the temperature for the Aluminum panel is linear as one would expect for a homogenous material. However, FGMs and TBCs produce nonlinear temperature profiles. Considering the TBC first, one can see that there exists a drastic change in temperature at just above the center of the panel thickness. The abrupt change in temperature can be explained by the fact that the TBC is bonded to the Aluminum substrate, creating a drastic change in
material properties (discontinuity). This is in contrast to the FGM as the temperature profiles are smooth with no abrupt changes in temperature.

The maximum in-plane stress in the middle of the panel for each case in the analysis can be seen in Figure 5. As with the temperature, the stress for the Aluminum panel is linear, with a large range in stress, while the TBC and FGM panels create nonlinear stress profiles. The TBC creates a drastic change in stress where the TBC is bonded to the Aluminum substrate. Furthermore, the FGM is able to reduce the stress concentration found in the TBC and also reduce the stress range, particularly with small values of n. Larger values of n (2 and 5 in this work) also reduce stress concentrations and stress range, but not to the level of n=0.2. Table 3 quantifies the change in stress in the middle of the plate on the outer layer. Examining Table 3, the FGM is able to reduce the maximum in-plane principle stress for the outer layer by up to 40% for n=0.2. Considerable reductions are also noted for FGMs with n=2 and 5. Additionally, the weight of the panel was lowered considerably by as much as 29%.

**Conclusions**

Functionally graded materials have been studied to determine the effect of continuously grading material properties of composites for high velocity and temperature aerospace applications. Volume fractions have been modeled to account for the change in volume fractions as a function of the thickness coordinate. Material properties were estimated using Voigt’s and Reuss’ bounds, while using conservative approaches to choosing appropriate bounds. Functionally graded materials have been modeled for use of finite element analysis by solving the coupled heat transfer and equations of motion. It was found that FGMs exhibit a
nonlinear temperature and stress variation through the thickness with a reduction in the temperature of the base material. Stress concentrations have been reduced by producing a smooth and uniform stress field through the thickness and in-plane directions. Furthermore, it has also been shown that a significant reduction in the weight of the panels is possible.
Figures

Figure 1: Steady state temperature vs. Mach number at 80,000-ft [7].

Figure 2: Visual effect of material grading for n=1.
Figure 3: Distribution of the ceramic volume fraction of the FGM.

Figure 4: Through-thickness temperature profile.

Figure 5: Through-thickness maximum in-plane stress.
### Tables

**Table 1: Material properties.**

<table>
<thead>
<tr>
<th>Layer</th>
<th>Base</th>
<th>TBC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Property</td>
<td>Aluminum</td>
<td>Zirconium</td>
</tr>
<tr>
<td>$E$ [GPa]</td>
<td>90</td>
<td>151</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>$\alpha$ [μm/m]</td>
<td>23</td>
<td>10</td>
</tr>
<tr>
<td>$k$ [W/m-K]</td>
<td>204</td>
<td>2.09</td>
</tr>
<tr>
<td>$\rho$ [kg/m³]</td>
<td>2,707</td>
<td>5,680</td>
</tr>
</tbody>
</table>

**Table 2: Panel weight.**

<table>
<thead>
<tr>
<th>$n$</th>
<th>Wt.</th>
<th>% difference from TBC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[kg]</td>
<td>[lbs]</td>
</tr>
<tr>
<td>Zirconium</td>
<td>3.79</td>
<td>8.35</td>
</tr>
<tr>
<td>Traditional TBC</td>
<td>3.13</td>
<td>6.89</td>
</tr>
<tr>
<td>FGM n=0.2</td>
<td>3.34</td>
<td>7.35</td>
</tr>
<tr>
<td>FGM n=2</td>
<td>2.51</td>
<td>5.53</td>
</tr>
<tr>
<td>FGM n=5</td>
<td>2.22</td>
<td>4.89</td>
</tr>
<tr>
<td>Aluminum</td>
<td>1.81</td>
<td>3.98</td>
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</table>

**Table 3: Changes in stress, outer layer middle of panel.**

<table>
<thead>
<tr>
<th>Position</th>
<th>Max Stress [MPa]</th>
<th>% difference from TBC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer layer</td>
<td></td>
<td></td>
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<tr>
<td>Traditional TBC</td>
<td>172</td>
<td>-</td>
</tr>
<tr>
<td>FGM n=0.2</td>
<td>31</td>
<td>-40%</td>
</tr>
<tr>
<td>FGM n=2.0</td>
<td>61</td>
<td>-31%</td>
</tr>
<tr>
<td>FGM n=5.0</td>
<td>133</td>
<td>-11%</td>
</tr>
<tr>
<td>Aluminum</td>
<td>353</td>
<td>51%</td>
</tr>
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</table>
References


